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AN ASSESSMENT OF VIRTUAL REALITY TECHNOLOGY FOR ASTRODYNAMICS APPLICATIONS

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Recent portable and affordable virtual reality (VR) devices may be a tipping point for the diffusion of immersive working environments. Our work focuses on an early assessment of modern VR technology for astrodynamics applications. The assessment is constructed by a review of VR-related works that are external to the typical astrodynamics community to facilitate cross-pollination of ideas. Next, the Johnson-Lindenstrauss lemma, together with a set of simplifying assumptions, is employed to analytically estimate the time-to-discovery within a dataset that is projected to lower dimensions. Finally, two astrodynamics applications are presented to demonstrate solutions that are primarily enabled by VR technology.

INTRODUCTION

Vision-based interaction with sets of multi-dimensional information mitigates the complexity of several applications in astrodynamics. For example, visual-based processes are key to understanding solution space topology for orbit mechanics (e.g., Poincaré maps),[1] formulating higher quality initial guesses for spacecraft trajectory optimization,[2] and investigating six-degree-of-freedom (6DOF) dynamics for proximity operations.[3]

Interactive visualization intervenes at different stages of a mission design process, as detailed by Stuart.[4] In fact, state-of-the art mission design software, including GMAT,[5] Copernicus,[6] Monte,[7] Satellite ToolKit (STK),[8] and FreeFlyer,[9] importantly relays on desktop computer graphics to easy simulation set up and facilitate analysis of the results. Researchers at Purdue University in collaborations with NASA Goddard Space Flight Center have demonstrated a visual-based framework, known as Adaptive Trajectory Design (ATD) toolkit, to enable rapid and intuitive end-to-end trajectory within the multi-body cislunar space.[2,10] The ATD experience demonstrates the importance of visual-based design to identify high-quality initial guesses for trajectory correction algorithms and to guide the transition from lower to higher fidelity models.[2] Davis et al. at a.i. solutions have internally developed a Deep Space Trajectory Explorer (DSTE) toolkit, a trajectory design interface for gravitation multi-body environments that is centered around visual interaction with Poincaré maps to identify desirable solutions.[11] Visualization of network graphs with AUTO software has facilitated study of dynamical connections between families of orbits within the circular restricted three-body problem (CR3BP).[12]

Visual-based processes work in synergy with optimization routines. Insight gained via human-machine interactions may guide trajectory design within dynamics that are too sensitive to be directly approached with gradient-based methods and optimization algorithms. Interactive visualiza-

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tion may also aid global optimization searches, enabling to recognize patters in locally-optimal solutions that facilitate hopping across local basins of convergence toward the global minimum. Anecdotally, the winning solution of the fifth Global Trajectory Optimization Competition (GTOC) has been discovered by visually inspecting the best solution returned by a set of evolutionary optimization algorithms.[13] The objective in the 5th GTOC is to visit the largest number of asteroids in the main-belt. Visual inspection of the optimizer solution has made obvious that an additional asteroid, one on-route of the current spacecraft trajectory, could have been added to the tour sequence. That addition has been critical in reaching an higher solution score and winning the competition. While interactive visualization powered by desktop computer graphics has already enjoyed an enormous success, the application of the Window-Icon-Mouse-Pointing (WIMP) paradigm to interactive visualization is challenged by the dimensionality of the target dataset: higher the dimension, the more complex is to maintain the representation intuitive. In fact, since the advent of modern computers, the capillary diffusion of heterogeneous, multi-dimensional data throughout astrodynamics has motivated the creation of virtual reality (VR) technology to facilitate scientific discovery. However, the high installation cost and complexity of operation for earlier systems,[14] such as multi-sided displays[15] and early VR visors,[16] may have prevented the adoption of VR technology by a large base of users in astrodynamics. The recent appearance of small, portable, and affordable devices may be a tipping point to advance astrodynamics applications via VR technology. Head Mounted Display (HMD) are becoming accessible to a massive audience, therefore potentially offering ubiquity of VR access. In some cases, HMDs represent a significant financial advantage over other solutions,[17] empower the user with data analysis capabilities equal or superior to legacy CAVE systems[18] and can reduce user stress compared to smaller desktop display.[19] In astrodynamics, more accessible VR equipment is inspiring the creation of immersive frameworks for mission design and astrodynamics applications. Currently, interfaces capable of VR visualization are, in fact, being developed for existing WIMP-based astrodynamics software.[4] Nonetheless, the tangible benefits for adoption of virtual reality frameworks are not yet fully understood and characterized in the context of astrodynamics applications. *What new opportunities virtual reality opens for astrodynamics? What applications would benefit the most from virtual reality frameworks?* To explore these and similar questions, our work offers an initial assessment of VR technology for astrodynamics applications. The assessment is constructed by a review of VR literature with elements that are external to the astrodynamics community to facilitate cross-pollination of ideas. Next, the Johnson-Lindenstrauss lemma, together with a set of simplifying assumptions, is employed to analytically capture the value of projecting higher-dimensional information to a given lower dimensional space. Finally, two astrodynamics applications that are being developed at Auburn University are presented. This final discussion is aimed to display solutions that are primarily enabled by VR technology.

LITERATURE REVIEW

We have reviewed a modest body of cross-disciplinary VR-related literature with the goal of consolidating knowledge that may inform astrodynamics applications.

Celestial mechanics, Henri Poincaré, and visual analytics. Celestial mechanics first, and astrodynamics later, often demand access to the solution of non-integrable dynamics. The demonstration of the non-integrability of the Three-body problem by Henri Poincaré is a prominent example. However, Poincaré also proposes a solution to the treatment of non-integrable dynamics. The solution is based on the Poincaré recurrence theorem. According to this theorem, if a dynamical system is volume-preserving and all orbits are bounded, then all trajectories return to a state arbitrarily close to their initial state after a finite time. The practical implication of Poincaré recurrence theorem is that non-integral dynamics could be explored by visually sampling the phase-space volume to identify patters of recurring states (i.e., Poincaré maps). Poincaré maps are just another example of how visualization has enabled significant breakthroughs throughout the previous century.[20] It is well understood that visualization is essential to bridge quantitative information with human intuition, fueling analytical reasoning and shortening the path to discovery.

Biological evidences. Empirical evidence supports the idea that several animal species, including humans, evolved their organisms to optimize the acquisition, processing and utilization of visual sensory input to operate in a three-dimensional space. Visual dominance in human information processing is a well known biological fact.[21] Visual dominance is the tendency of visual inputs to occur more rapidly and more frequently in perceptual and memorial reports than other senses. In addition, controlled experiments on selected animal species[22–24] seem to demonstrate visomotor coupling. Visomotor coupling is the reciprocal dependence between visual inputs and movement-related states in environment information processing and body motion control.

Benefits of immersion in visual analytics. Immersion may bring advantages to visual data analysis. Immersive visualization of spatially organized information (also known as memory palaces) via head-mounted displays (HMDs) provides a superior memory recall ability compared to desktop visualization of the same set of information.[25] Human interactions with data structures also improve with immersive visualization over traditional desktop applications.[26,27] In fact, documented evidence shows that virtual reality yields higher user satisfaction and depth of insight if data are multi-dimensional and contain spatial structures.[28–30] Immersion via stereoscopic displays paired to a high field of regard can facilitate both large-scale spatial judgment tasks, such as understanding of volumes and isosurfaces,[31] and small-scale spatial judgement, such as examining the connectivity of large networks.[32] On such premises, VR interactive interfaces have been proposed across different disciplines, from visualization of vector fields in physics and math,[33] to aerodynamics research with the NASA Virtual Wind Tunnel,[34] to programming and control of robotic systems.[35–37] Immersion may be employed to establish a workflow that facilitates collaboration when the interdisciplinary discussion and integration of heterogeneous raw data is required for evaluation and verification of research results.[38,39] By removing clutter, immersion may also increase focus when conducting analysis or taking decisions.[17]

Challenges of immersion in visual analytics. Caution is required in developing VR applications for data analysis, as advantages offered by immersion may be rapidly washed away by ill-informed user-interaction solutions. Using semi-randomized 3D scatter plots, research demonstrates the importance of plot navigation and rotation-scale-alignment to efficiently discover data patterns.[40] Collaborative data analysis among multiple users is not necessarily more effective in VR than traditional media, if the VR experience is not supported and orchestrated by proper tools and interaction mechanisms.[41,42] The co-presence of spectators or collaborators who are not immersed in the VR environment may create a sense of embarrassment in VR users.[43] Participants who are not immersed and VR users may lack of mutual understanding of their experience.[43] Therefore the utilization of VR applications may become challenging within shared or public spaces. Furthermore, although more enjoyable, VR applications may have steepest learning curves than traditional user interfaces.[44] While VR may enable super-human interactions with the environment, such interactions may not correspond to an increase of performance in completing a given task. For example, a study finds that simultaneously operating more than two hands (a.k.a. the human octopus) may not necessarily yield a higher success rate for a given manipulation task.[45]

Interaction with VR environments. The user interaction paradigm may be an important drive for the success of a VR user application.[46,47] User interaction interfaces are characterized by several properties, including usability, easy of learning, manipulation mode, measure time, presence, and naturalness of the input mode.[48,49] Both usability and easy of learning are directly related to intuitiveness of the user interface, which is known to be one of the most important factor in user interface design.[50] Intuitiveness is often correlated with the naturalness of interaction. Commonsense often dictates that emulating real-word physics results in more intuitive interactions for the general end-user. Manipulation mode describes which tasks can be accomplished through the interface. Measured time is the temporal duration to complete a given task using the interface. Presence is the conscious sense of belonging to the virtual world.[51,52] Presence is a subjective feeling which is hard to measure; therefore, presence is better assessed as overall satisfaction of the user with the interface. There currently exist several mechanisms for user interface interactions, including 2D interactions, gesture-based interactions, speech input, and 3D interactions. 2D interactions are often equivalent of WIMP interfaces. Adopting a WIMP approach in virtual reality requires to be cognizant of challenges associated with: 1) accuracy of selection using a directional beam; 2) text readability on low resolution devices; 3) preservation of the feeling of presence and immersion. Gesture-based interactions utilize body gestures or movement of body parts, including finger tracking and eye gazing, to control the VR environment. At the current stage of development, gesture-based interactions are a potential source of frustration for the user due to limited tracking accuracy or complexity of the enabling hardware. For example, visual tracking of finger gestures may display usable tracking accuracy only within a narrow field of view. Improving tracking accuracy with the addition of sensors results into bulkier and more expensive control devices.[53] However, if these challenges are overcome, hand gesture control has a high potential for developing very intuitive VR interfaces.[54] Speech input utilizes voice commands to interact with the virtual environment. Among the types of interaction considered, speech input has been shown to be the most intuitive for the users, resulting in quicker task completion (under controlled experiment conditions).[49] However, important challenges may arise in recognizing uncommon vocabulary and expressions, such those that might be utilized in performing complex tasks. Moreover, users might be resistant to use speech input in a public environment, such a shared office space. 3D interactions refer to the utilization of a device such hand-held controllers, joystick or haptic gloves to interact with 3D objects in the VR scene in a form that emulates physics. 3D interactions can be extended to distant objects using selection beams and go-go interaction techniques. Possible user challenges in adopting 3D interaction techniques include the lack of physical feedback[19] and asyncronies between visual and haptic feedback.[55] Technical solutions to provide haptic feedback range from controller vibration to haptics gloves. Haptic gloves are often bulky, expensive and has low technology readiness level. Another challenge with 3D interactions is the higher number of degrees of freedom that can be mapped to user actions. An increase of freedom may challenge both the input device as well as the user.[56]

Current applications. Virtual Reality is rapidly gaining a foothold in applications that involve the acquisition of procedural or declarative knowledge, social interactions among individuals, and entertainment. By emulation, VR environments may reduce the cost of training and enable to safely replicate potentially harmful situations for practice.[57] Then, it comes at no surprise that military and medical applications are one of the earliest adopters of VR technology. In the military, reported studies present applications such as combat simulation, warfare operations, and mine/bomb training.[58–60] In the medical field, VR has been utilized to practice surgical procedures and to visualize tumors, or other medical information.[61,62] Because dangerous situations may be replicated in a safe and controlled matter, VR is also employed in the therapeutic treatment of anxiety, disorder, phobia.[63–66] Together with training, entertainment is where the private sector foresees the currently most accessible path for return of investment. Applications such as gaming, social media, and virtual tourism are at the forefront of VR content development.[67–69]

Immersive learning. In education, learning in virtual environment may increase student's engagement, competence and skills over traditional pedagogy.[70] In the physical dimension, immersive learning allows natural interactions with content that can only exist, or that is more practical to present, in a computer-generated environment.[71–73] In the cognitive dimension, immersive learning enables spatiotemporal alignment of information and the creation of other aids that foster understanding.[73–75] In the contextual dimension, immersive learning facilitates collaborative learning and experiences that are personally meaningful.[76] Example of VR applications for education include tools for spatial algebra, visualization of 3D geometries, museum-like presentation of complex function graphs, and computer-aided design.[77] However, we should acknowledge a general lack of experience in using VR lab among educators[78] and the existence of multiple uncertainties in selecting the appropriate methodology to incorporate VR content into an existing curriculum.[79]

Gaps in current applications. While popular in specific fields of application and among earlier adopters, virtual reality may also be received with skepticism due to lack of meaningful content. Such lack of content seems predominant in applications that engage more directly with analytical thinking. For example, a recent study reports that only 12% of VR learning content is related to analytical and problem solving skills[80] (compared to 60% of the content related to procedural and declarative knowledge). A query on Web of Science for the topic "virtual reality" returns search results that are summarized in Figure 1. Figure 1 may highlight that most VR research is still concentrated in VR hardware and software development, with less consideration of end-user scientific applications (except for the medical field). Similarly, while the Perksins Coie 2019 "Augment and virtual reality survey report technology" points to the lack of content as the second most important barrier to consumer adoption, the Perksins Coie 2020 report does not enumerate science (excluded medical applications) and engineering (excluded military applications) among the fields that are perceived to offer the highest return of investment. These observations align with technology acceptance studies,[81] recognizing that virtual reality acceptance is highly correlated with the presence of meaningful, possibly innovative, content. Easy to use judgment may also link to the perceived usefulness (and acceptance) of VR applications; users may measure how VR technology is easy of utilize against their past experience with traditional interfaces.[82] Finally, the high cost associated with building VR experiences may be an additional barrier to VR content acceptance.[82]

APPLICATION OF THE JOHNSON-LINDENSTRAUSS LEMMA

The mathematical theory of projections may enable to abstract the problem of insight discovery within a set of multi-dimensional data that is projected onto a lower dimensional space. Via the theory of projections, it may be possible to analytically capture some of the advantages in using higher dimensional working environments, such as virtual reality. The theory of projections is based on the Johnson-Lindenstrauss lemma,[83] which states

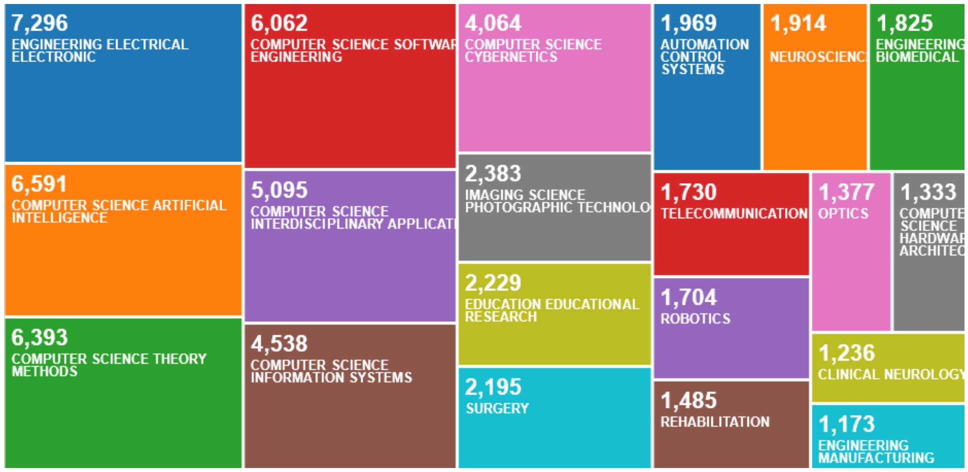


Figure 1: Search tree from Web of Science query for topic "virtual reality". Numbers indicate reference count in the corresponding field.

For any $0 < \varepsilon < 1$ and any integer $n > 1$, let k be a positive integer such that $k \leq k_0$ with $k_0 = C\varepsilon^{-2}\ln(n)$, where C is a suitable constant. Then, for any set V of n points in $\mathbb{R}^d$, there exists a map $f : \mathbb{R}^d \rightarrow \mathbb{R}^k$ such that for all $u, v \in V$,

$$(1-\varepsilon)||u-v||^2 \leq ||f(u)-f(v)||^2 \leq ||(1+\varepsilon)||u-v||^2$$

The Johnson-Lindenstrauss lemma states that, if the dimension k is greater than a minimum value, there always exists a projection from $\mathbb{R}^d$ to $\mathbb{R}^k$ that preserves pairwise distances, $||f(u)-f(v)||^2$, within a given error, ε, relative to the original value, $||u-v||$. Preservation of pairwise distances among points in the dataset may be employed as a measure of preservation of information after projection to a lower dimensional space. However, the minimum space dimension, k_0, grows quadratically with the inverse of the tolerable error, ε. For small error values, ε, the minimum space dimension, k_0, is in the order of 10^2. Therefore, the Johnson-Lindenstrauss lemma guarantees the existence of information-preserving projections onto lower dimensions only for original dataset starts at very high dimensions. Figure 2 visually describes the idea that typical astrodynamics problems are associated with state space dimensionality in the order of 10^1-10^2. In this range, the preservation of pairwise distances - when projecting to lower dimensional spaces - cannot be guaranteed in the Johnson-Lindenstrauss sense. Nonetheless, projection to lower dimensional spaces is a necessity to render data accessible to human operators. Even when astrodynamics problems are solved by black-box algorithms (which can operate in any state dimensionality), human intuition may be necessary for the identification of high-quality initial guesses, formulation of correct assumptions, and interpretation of results. Although it has been originally developed to facilitate nearest-neighbor search algorithms in very high dimensions, the theory of projections stemming from the Johnson-Lindenstrauss applies to any state dimensionality.

By utilizing the theory of projections in lower dimensions, we can analytically describe the time-to-discovery as a function of projection space dimension. The Johnson-Lindenstrauss lemma yields the following random projection theorem[83]

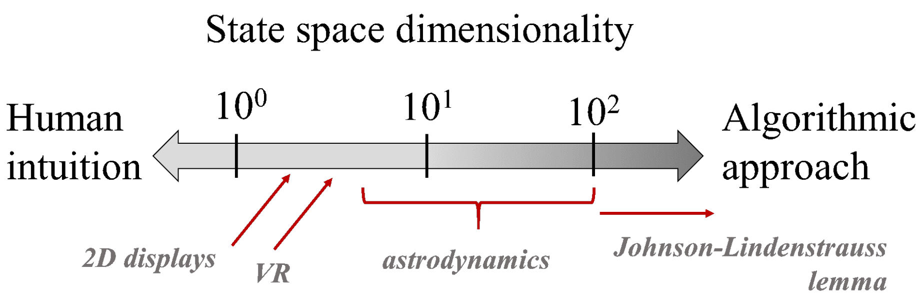
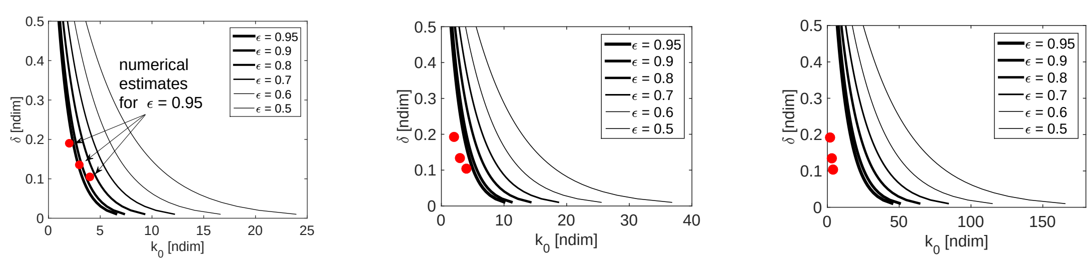


Figure 2: Typical state space dimensionality of astrodynamics problems.

For any real number $0 < \varepsilon < 1$, real number $\delta < 1/2$ and positive integer d, there exists a random matrix T of size k x d, such that for $k > k_0$ with $k_0 = C\varepsilon^{-2}\ln(1/\delta)$ and for any unit length vector $x \in \mathbb{R}^d$, then

$$P\left\{\left|||Tx||^2 - 1\right| > \epsilon\right\} < \delta \tag{1}$$

Equation (1) supplies an upper bound, δ, to the probability of vector length errors greater than a given threshold, ε, after the reference vector is randomly projected to a lower dimension. The constant value, C, is not specified in the original theorem. Different authors suggest a value of $C \approx 9$,[84] $C \approx 8$,[85] and $C \approx 2$ for practical applications.[86] Figure 3 displays mapping of k_0 values to the upper probability bound δ as a function of different ε and C values. Red markers in Figure 3 indicate estimate probabilities for $\left|||Tx||^2 - 1\right| > \epsilon$ from Monte Carlo simulation of 50000 random projections of an arbitrary six-dimensional vector to a lower dimensional space of dimension k_0 (three values are considered $k_0 = 2, 3, 4$).



(a) $C = 1.3$ (b) $C = 2$ (c) $C = 9$

Figure 3: Mapping dimesion of the projection space, k_0, to upper probability bound, δ.

To estimate the time-to-discovery, we assume (admittedly, not comprehensive of all applications) that a discovery is made within the projection space only when the human operator is able to tell apart all points of interest that are, in fact, distinct in the fully-dimensional dataset (i.e., pairwise distance in the projection space is greater than a minimum value).[87] Assume the reference vector length in Eq. (1) represents the pairwise distance between two distinct points within the fully-dimensional dataset. An overlap event O_i occurs after projection when the resulting pairwise distance error, $\left|||Tx||^2 - 1\right|$ is greater than a given threshold, ε. Overlap events occur with probability $P(O_i)$

bounded by Eq. (1) for random projections. Then, the probability of not observing overlap after random projection for the selected pair of points is

$$P(\bar{O}_i) = 1 - P(O_i) \tag{2}$$

If the dataset, D, comprises N points of interest, there exist $N(N-1)/2$ unique pairs of interest. Assuming the projection of each pairwise distance vector to be an independent event (e.g. the reference dataset is randomly generated), the probability that none of the pairs overlaps after a single random projection trial is

$$P(\bar{O}_t^D) = \bigcap^{N(N-1)/2} P(\bar{O}_i) = \prod_i^{N(N-1)/2} P(\bar{O}_i) = (1 - P(O_i))^{N(N-1)/2} \tag{3}$$

Then the probability that at least one pair overlaps after a single random projection trial is

$$P(O_t^D) = 1 - P(\bar{O}_t^D) = 1 - (1 - P(O_i))^{N(N-1)/2} \tag{4}$$

and the probability of having at least one pair overlap in each consecutive random projection trial is

$$P(O^T) = \bigcap_t^m P(\bar{O_t^D}) = \prod_t^m P(O_t^D) = \left(1 - (1 - P(O_i))^{N(N-1)/2}\right)^m \tag{5}$$

where trails are assumed independent events and m denotes the total number of trials. "Discovery" for a series of m trials is, then, defined as the occurrence of at least one projection that maintains pairwise distances within the given tolerance for all pairs of interest in the dataset. The probability of discovery, $P(D^T)$, for m successive trials with no memory is complementary to probability, $P(O^T)$, of having at least one pair overlap in each consecutive random projection trial, therefore

$$P(S^T) = 1 - P(O^T) = 1 - \left(1 - (1 - P(O_i))^{N(N-1)/2}\right)^m \tag{6}$$

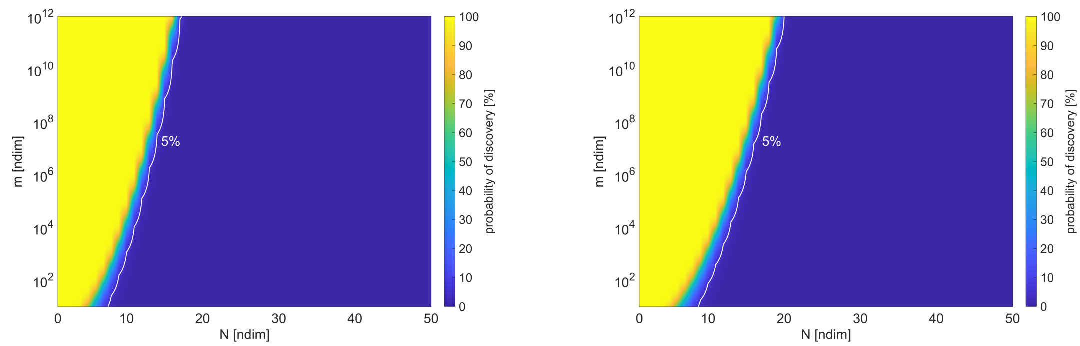
In the worst case scenario, when the base probability of overlap, $P(O_i)$, is equal to the upper-bound value predicted by the random projection theorem in Eq. (1)

$$P(O_i) = \delta = e^{-\dfrac{k_0 \varepsilon^2}{C}} \tag{7}$$

the probability of discovery may be written as a function of the dimension of projection space, k_0, and error threshold that defines an overlap, ε

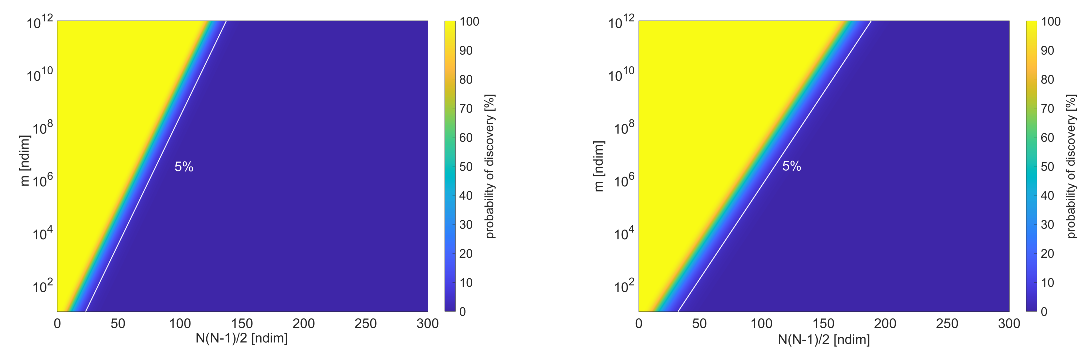
$$P(D^T) = 1 - P(O^T) = 1 - \left(1 - \left(1 - e^{-\dfrac{k_0 \varepsilon^2}{C}}\right)^{N(N-1)/2}\right)^m \tag{8}$$

Using Eq. (8), we may estimate the probability of discovering that N points in $\mathbb{R}^6$ are distinct when observing successive random two-dimensional and three-dimensional projections of the fully-dimensional dataset with no memory. For a value $\varepsilon = 0.95$ (i.e., the separation distance between two points after projection is reduced to 5% of its original value) and $C = 1.3$, the probability of overlap for an individual pair, $P(O_i)$, is about 20% for $k_0 = 2$ (two-dimensional projection space) and 15% for $k_0 = 3$ (three-dimensional projection space). The selected estimates for $P(O_i)$ values well align with the upper bound, δ, estimates displayed in Figure 3. For the selected $P(O_i)$ values, Figures 4 displays the probability of discovery, $P(D^T)$, as a function of the number of trials, m, and number of points, N. Figures 5 displays the same information, but as a function of the number of pairs, $N(N-1)/2$. Assuming that a display screen corresponds to a two-dimensional projection space whereas a virtual reality environment is a three-dimensional projection space, then, Figure 4 and Figure 5 display how the additional dimension offered by a VR environment may shorten the path to discovery. For a given number of points or pairs, fixed the desired probability of discovery, the number of trials to achieve a desired probability of discovery may be orders of magnitude smaller when using three-dimensional projections.



(a) 2D projection ($k_0 = 2$, $P(O_i) = 0.2$ (b) 3D projection ($k_0 = 3$, $P(O_i) = 0.15$

Figure 4: Probability of discovery as a function of number of points of interest, N, and number of trials, m



(a) 2D dimensional projection ($k_0 = 2$, $P(O_i) = 0.2$ (b) 3D projection ($k_0 = 3$, $P(O_i) = 0.15$

Figure 5: Probability of discovery as a function of number of pairs of interest, $N(N-1)/2$, and number of trials, m

COORDINATION OF VISUAL UNDERSTANDING WITH COMPLEX TASK

In addition to facilitating retention of information and interpretation of high-dimensional data, virtual reality has the potential to ease the execution of tasks that require visual understating of the operational environment. In the following, we briefly present two astrodynamics applications where the execution of complex tasks is coordinated with a visual understanding of the corresponding problem.

Trajectory design via the act of drawing

The idea that user-drawn curves may be mapped to feasible spacecraft trajectories is inspired by Schlei's work at Purdue University.[88,89] Schlei proposed this idea working mainly with two-dimensional screens. However, when users attempt to draw three-dimensional curves through a display screen the interaction with the third dimension is a cumbersome process. As a result, ambiguity remains in the interpretation of the third dimension in curves that users draw through a display screen. A fully immersive virtual reality framework surpasses this possible limitation, enabling the users to draw in real three-dimensions. In the context of the circular restricted three-body problem (CR3BP), we have developed a prototype VR framework to investigate how user-drawn curves may be mapped to feasible spacecraft trajectories. The prototype concept is visualized in Figure 6. The framework is based on a front-end interface developed in Unity3D and on a back-end collocation algorithm to map user drawings to spacecraft trajectories. A nonlinear optimizer is employed because the Earth-Moon CR3BP system dynamics are nonlinear and coupled. The orthogonal collocation based optimizer is employed for finding the closest natural trajectory to the user-drawn trajectory. The theory and the equations relevant to the optimizer are presented by Grebow and Pavlak.[90,91] The optimization scheme from Pritchett is implemented in our framework.[92]

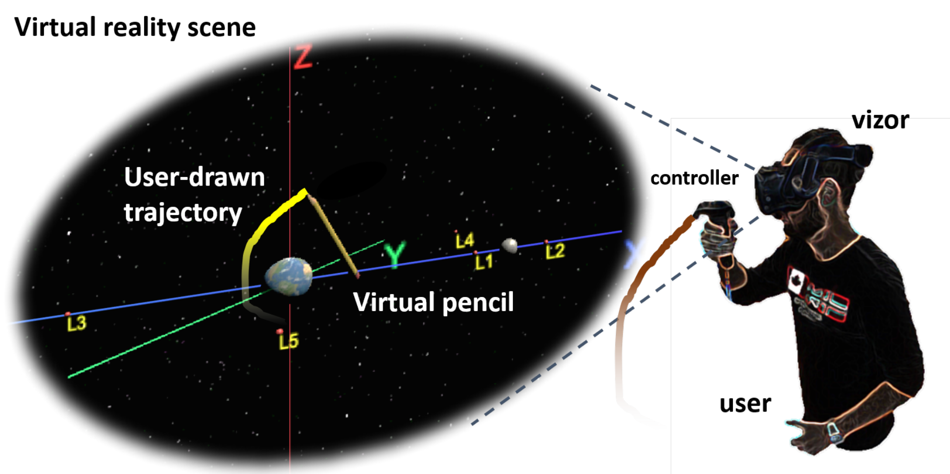


Figure 6: VR prototype concept to enable trajectory design via the act of drawing.

The user input comprises a position dataset (extracted from the curve drawn in configuration space) and an energy measure for the CR3BP, J, called the Jacobi constant. To easy convergence of the optimizer, an educated estimate for the velocity states, $\mathbf{v}_i$, is derived from the user input data and supplied to the optimizer as initial guess. At the sampled position states, $\mathbf{p}_i$, along the trajectory, a central difference scheme is used for computing the velocity direction, $\mathbf{v}_i/||\mathbf{v}_i||$, and a forward difference scheme is used for computing the time-of-flight for the trajectory arc in-between the sample position points, ΔT_i. These are given by the equations,

$$\frac{\mathbf{v}_i}{||\mathbf{v}_i||} = \frac{\mathbf{p}_{i+1} - \mathbf{p}_i}{||\mathbf{p}_{i+1} - \mathbf{p}_i||} \quad \text{with} \quad \mathbf{p}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \tag{9}$$

$$||\mathbf{v}_i|| = \sqrt{x_i^2 + y_i^2 + \frac{2(1-\mu)}{d_i} + \frac{2\mu}{r_i} - J} \tag{10}$$

$$\Delta T_i = \frac{||\mathbf{p}_{i+1} - \mathbf{p}_i||}{||\mathbf{v}_i||} \tag{11}$$

where the subscript i denotes the current sample position point, μ is the CR3BP mass parameter, and d_i and r_i denote instantaneous distances of the spacecraft from the larger and the smaller primaries respectively.

Using a controlled experiment, we study how quality of the initial drawings impacts the ability to recover a desired spacecraft trajectory. A reference L_2 halo trajectory is presented to the users in the VR simulator for the Earth-Moon system to trace in its original location. Various users trace the reference L_2 halo trajectory in-place in real three-dimensional space and save the drawing. Two samples of user-drawn trajectories are displayed in Figure 7. The saved drawings are collected by the authors to compute the statistics for distortions, to test the robustness of the optimizer and to conduct feasibility analysis to find closest feasible trajectories.

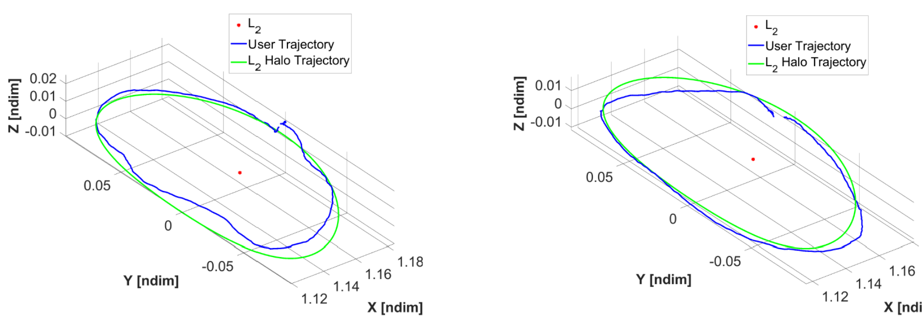


Figure 7: Sample user drawing and L_2 halo reference trajectory in the CR3BP Earth-Moon system.

In this experiment a total of 103 saved user-drawn trajectories are collected. The collected user trajectories are processed by the optimizer to find the nearest feasible trajectory. For each user-drawn trajectory, the mean Euclidean-distance error from the L_2 halo reference trajectory defines a measure of solution quality. A quality measure value of 0.005 non-dimensional units, or approximately 2000 kilometers, is applied to the converged trajectory as a threshold for accepting or rejecting the solution (i.e., solutions with as quality measure value above the threshold are rejected). With such criteria, the optimizer was able to find acceptable feasible trajectories for 99 of the user-drawn curves. Each plot within Figure 8 portrays three different trajectories in the non-dimensional space of the Earth-Moon rotating frame: the curve in red color is the reference L_2 halo trajectory, the curve in blue is the user drawing and, finally, the faint green curve is the solution trajectory found by the optimizer. The left plot in the figure is showing a converged trajectory where the mean-distance-error of the solution from the L_2 halo trajectory is the minimum (515 km) and the right plot is showing a converged trajectory with maximum mean-distance-error (1859 km) among the accepted 99 solutions.

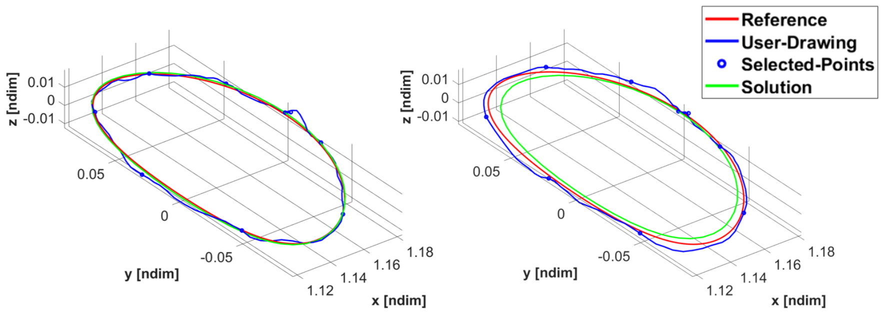


Figure 8: Sample user drawing, L_2 halo reference and converged trajectories.

For each drawing, by establishing normal correspondence with the reference L_2 halo trajectory, shape distortions relative to the reference may be computed according to classical continuum mechanic theory. Using an affinity transformation that renders roto-translational motion and linear deformation of the user drawing relative to the true, reference trajectory, shape distortions are decomposed into four fundamental modes: rigid translation, rigid rotation, normal strain, and shear strain. The mean and standard deviation for each distortion mode are computed from the user drawings sample. Then, the reference halo trajectory is distorted randomly in each of the distortion fundamental modes using the recorded probability distributions. Next, the distorted trajectory is provided as the initial guess to the optimizer to find the closest feasible periodic solution. One hundred such distorted trajectories are processed by the optimizer in a Monte-Carlo analysis for each of the fundamental modes of distortions. Table 1 lists the results obtained in the Monte-Carlo analysis. Labels x, y, z refer to the Cartesian directions defined by the CR3BP rotating frame: x is aligned with the primaries, z is orthogonal to the primary orbit plane, and y completes are right-handed coordinate system. The Converged Count column in this table indicates that the optimizer responds generally well to foreseeable rigid translation and strains in the y direction. The selected optimizer is, instead, less robust to rotational and strain distortions in the x, z directions.

Table 1: Distortion Analysis Monte Carlo Results

Distortion Mode	Converged Count	Distortion Mode	Converged Count
Rigid translation, x	89	Rigid rotation, 3-2-1 Euler body sequence	79
Rigid translation, y	98		
Rigid translation, z	100		
Normal strain, x	62	Shear strain, x-y	74
Normal strain, y	99	Shear strain, y-z	89
Normal strain, z	69	Shear strain, z-x	71

The ability to draw curves in a real three-dimensional environment and recover feasible spacecraft trajectory is one of new applications that may be enabled by VR technology. However, it is evident that further research is warranted to seamlessly integrate existing trajectory design procedures and VR interfaces into an usable immersive computing framework.

6DOF VIRTUAL REALITY INTEGRATED SPACECRAFT PROXIMITY OPERATIONS SIMULATOR

Immersive computing frameworks may also improve accessibility and visualization for the control of spacecraft in proximity operation scenarios, including docking, repairs, maintenance, and a multitude of other applications. For instance, a Goddard-developed VR simulation environment is in the working to assist engineers with hardware integration and testing for future missions, including Restore-L.[93] As a starting point to assess the potential of virtual reality for proximity spacecraft operations, we are developing an immersive six-degree-of-freedom (6DOF) simulation environment. In this program, the virtual reality headset allows the user to pilot the spacecraft from a 3rd person perspective. As the HMD is activated, the room the user occupies transforms into a virtual space. This virtual scene offers a third-person view of a scaled space vehicle and allows the user to walk around the vehicle surroundings. The user holds controllers that allow for a robust yet simplistic control scheme from which to maneuver the spacecraft. Control inputs from the user are fed into a program which translates said motion into a simulated reaction control system (RCS) thruster output, to move the spacecraft according to the users' preference.The left-hand controller direct motion along the three Cartesian axes, allowing the vessel to move up, down, right, left, forward, and backward, whereas the right-hand controller imparts rotation about the three body axes, allowing the vessel to pitch, roll, and yaw. Vessel's movements depends on the direction and magnitude of controller displacement. For example, moving the left controller to the left produces an RCS thrust in that direction, proportional in magnitude to the controller's displacement. The initial scenario chosen to be the focus of this project is a docking operation between an Orion Capsule and the Lunar Gateway. Approximate models of both the Orion Capsule and the Lunar Gateway are created using SolidWorks CAD software and exported as a mesh to Blender, which is then employed to create usable 3D assets for Unity3D. An approximate, geometry-based moment of inertia tensor for both objects may be estimated with SolidWorks. Such estimates are employed during the simulation of rotational dynamics.

This simulator is created using Unity3D, a popular platform for computer game development. The program can interpret C# scripts, ones that may be employed to customize simulation environment dynamics and record control inputs from the pilot. SteamVR provides the necessary bridge between the VR hardware and Unity3D, allowing Unity3D and C# scripts to read control movements. All of these programs together make up the basis of the proximity operations simulator. Developing an immersive computing framework for proximity operations may require to design and refine the simulator's physics engine. Unity3D features a native physics engine for rigid bodies, but it is not designed to be physically accurate for proximity dynamics simulation. Implementing a more accurate physics engine may require to rebuild the dynamics simulator from the ground up in C#. Note that, it may also be necessary to manually recreate matrix operations, cross products, and numeral propagation methods, when coding in C#. Within the simulator, the following differential equations describe spacecraft 6DOF motion

$$\begin{cases} \dot{\underset{\hat{i}}{\mathbf{x}}} = \underset{\hat{i}}{\mathbf{v}} \\ \dot{\underset{\hat{i}}{\mathbf{v}}} = \underset{\hat{b}\cdot\hat{i}}{[A]}^T \frac{1}{m} \underset{\hat{b}}{\mathbf{F}} \\ {}^{\hat{i}}\dot{\mathbf{q}}^{\hat{b}} = \frac{1}{2}[\Omega]\, {}^{\hat{i}}\dot{\mathbf{q}}^{\hat{b}} \\ {}^{\hat{i}}\underset{\hat{b}}{\omega}^{\hat{b}} = \underset{\hat{b}}{[I]}^{-1} \left(\underset{\hat{b}}{\mathbf{M}} - {}^{\hat{i}}\underset{\hat{b}}{\omega}^{\hat{b}} \times (\underset{\hat{b}}{[I]}\, {}^{\hat{i}}\underset{\hat{b}}{\omega}^{\hat{b}}) \right) \end{cases} \tag{12}$$

with

$$\underset{\hat{b}\cdot\hat{i}}{[A]} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \quad , \tag{13}$$

and

$$[\Omega] = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{14}$$

In Eq. (12) $\underset{\hat{i}}{\mathbf{x}}$ and $\underset{\hat{i}}{\mathbf{v}}$ denotes, respectively, the inertial position and velocity vector of the spacecraft, expressed in an inertial frame; $\sum \underset{\hat{b}}{\mathbf{F}}$ is the total external force acting on the spacecraft and written in a body-fixed frame. The resultant force, $\underset{\hat{b}}{\mathbf{F}}$, may include contributions from gravity exerted by multiple attractors and RCS thrusting actions. The quaternion vector ${}^{\hat{i}}\dot{\mathbf{q}}^{\hat{b}} = [q_1,\, q_2,\, q_3,\, q_4]^T$ describes the orientation of the spacecraft body-fixed frame relative to the inertial frame; the matrix $\underset{\hat{b}\cdot\hat{i}}{[A]}$ is the direction cosine matrix of the body-fixed frame relative to the inertial frame. The vector ${}^{\hat{i}}\underset{\hat{b}}{\omega}^{\hat{b}} = [\omega_1,\, \omega_2,\, \omega_3]$ denotes the angular velocity of the body-fixed frame relative to the inertial frame, expressed in body frame coordinates; the matrix $\underset{\hat{b}}{[I]}$ is the spacecraft inertia tensor and vector $\underset{\hat{b}}{\mathbf{M}}$ denotes the total external torque applied to the spacecraft, both expressed in the body-fixed frame. The resultant torque may include contributions for the gravity gradient exerted by multiple attractors, RCS controlling actions, and solar radiation pressure. Differential equations in Eq. (12) are integrated using a Runge-Kutta 4th order approximation with fixed time step equal to 0.02 seconds. At each simulation frame, the input from the controller is interpreted to determine any possible RCS control force or moment that the pilot applies to the spacecraft. A quaternion description of attitude kinematics in Eq. (12) is preferred over Euler angles to avoid gimbal lock. As we continue developing our VR simulator for 6DOF proximity operations, the first next step is the implementation of relative spacecraft orbit dynamics in multi-body environment and the incorporation of perturbing torques, such those exerted by gravity gradient, solar radiation pressure, and thrust vector misalignment.

Virtual reality may offer a lower entry barrier to investigations that require observing and controlling spacecraft motion in a visually realistic three-dimensional environment. In particular, VR

6DOF simulators may inform design and operations processes that require human supervision because full robotic automation is still challenging. Such as design or operations scenario may include extravehicular repairs, contingency operations, or intercepting / collecting uncooperative objects.

DISCUSSION

From our literature review, it appears that the application of immersive computing using VR technology is very diverse, but also sparse (with perhaps the exception of application clusters within training and entertainment). That may indicate that VR applications are at an early stage of evolution. Studies on human-machine interaction factors display the beneficial effects of adopting VR interfaces in visual analytics, but results are often obtained through simplified application scenarios. Factors that may arise within subject-of-matter applications, such as astrodynamics, are not yet understood. Waiting for the field to mature and stronger evidence to emerge, an early assessment of VR technology for astrodynamics must be conducted holistically. Across the applications surveyed, a few factors seem to converge as the common denominator that defines effective use of VR technology. Immersive computing via VR may increase productivity and/or shorten the path to insight discovery under the following circumstances:

- ☐ The target process relies on visual-based human-machine interactions.
- ☐ The target content is cheaper, safer or only exist in virtual space.
- ☐ The target process will benefit from better recall of information that can be spatially organized.
- ☐ Target data are multi-dimensional and heterogeneous.
- ☐ Target data structures are volumetric or spatial.
- ☐ The VR interface is equipped with well-designed scene navigation controls.
- ☐ Co-presence of multiple users in the scene is supported by well-designed collaboration tools.
- ☐ The target application can afford to discount a longer learning curve for the end-user.

One of the perceived strengths of immersive computing using VR is the additional dimension available to display information. The application of the Johnson-Lindenstrauss lemma may facilitate the identification of advantages and limitations associated with increasing the dimensionality of the space that receives the projection of higher-dimensional information. Referring to Figure 4, we may interpret the number of trials as a measure for the time-to-discovery; then, we observe that for a fixed probability of discovery, the time-to-discovery may be an order of magnitude lower when using 3D projections (such as in a immersive, virtual environment) instead of 2D projections (such as on a computer screen). For example, considering 10 points of interest and a 95% probability of discovery, the number of trials is approximately 10^5 when using 2D projections, and approximately 10^4 when using 3D projections. If we further assume that the cost of generating a new random projection is 0.25 seconds, which is equal to the average reaction time to a visual stimulus for humans,[94] 10 points of interest require 7 hours to achieve a 95% probability of discovery using random projections to a 2D space, 42 minutes using random projections to a 3D space. Fixed the probability of discovery, the order of magnitude for the number of trials grows quadratically with the number of points of interests and linearly with the number of pairwise distances of interest. From Figure 4 and Figure 5, it is also evident that for a modest number of points of interest (≈ 25), a random projection that preserves pairwise distance for the point of interests may not be possible in a practically finite time. This observation may imply that if the number of points of interest is sufficiently high, the advantage of projection to a 3D space over a 2D space may not be tangible during practical applications. However, such deduction is limited by the assumption of a memory-less observer in our application of the Johnson-Lindenstrauss lemma to derive the probability of discovery.

Foreseeing the benefits, experimentation and development of VR applications starts as early as computer graphics become possible. Earlier applications of immersive computing have encountered mixed fortunes, most never achieving an adoption scale that is comparable to that of certain computer desktop platforms for science and engineering. The lack of mass adoption may be attributed to hardware and operation costs for early HDM and CAVE systems. As HDMs become consumer electronics products, ubiquitous access to immersive computing may be a reality in the near future. Yet, the complexity of developing the software infrastructure to support VR applications remains a current barrier to a more capillary diffusion. In both applications developed in our lab, creating the graphical interface and interactions with the VR environment has been an additional burden, especially when compared to ready-to-use desktop software. In addition, the lack of basic scientific libraries in scripting languages that interface with the game engine may be an additional obstacle. Possible workarounds include developing in-house libraries or communication channels between the game engine and external computer programs. Finally, incomplete theoretical understanding of human-machine interactions in immersive computing for astrodynamics should also be considered. For example, within the immersive computing trajectory design framework under development, the convergence to feasible solutions may be driven by understanding the delicate interplay between user behavior (e.g., distortions present in user-drawn curves) and underlying dynamical structures.

In conclusion, our experience with VR applications for astrodynamics points to the fact that immersive computing may be best employed when data visualization is complemented by active, dynamic interactions with the environment. Rather than limiting immersive computing to an interactive analysis of static data, the user may be directly involved in a dynamic data generation process through interaction with the VR environment. Another advantages anecdotally observed, but also in line with published studies, is the ability of immersive computing to transform astrodynamics concepts into a tangible experience for novice users. Therefore, VR environments may easily become a tool for astrodynamics education, in academic settings or otherwise.

FINAL REMARKS

Over the last century, virtual reality has progressed at a steady pace. Current VR applications are diverse, but also sparse and undergo different fortunes. Benefits are not always tangible or transferable to new applications. This characteristic may be partially attributed to the subjective nature of human-computer interactions. While immersive computing is setting a foothold in fields such as training and entertainment, experimentation is on going within science and engineering applications. It is too early to determine whether the next-century astrodynamics will be performed through immersive interfaces, or immersive computing will remain a niche technology. As discussed in this study, evidences are promising and begin to delineate preferred directions of development, but they are not conclusive. Finally, human-computer interaction factors may play a more important rule in astrodynamics applications that utilize immersive computing.

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